Pauli Check Extrapolation for Quantum Error Mitigation

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***Abstract*—Pauli Check Sandwiching (PCS) is an error mitigation scheme that uses pairs of parity checks to detect errors in the payload circuit. While increasing the number of check pairs improves error detection, it also introduces additional noise to the circuit and exponentially increases the required sampling size. To address these limitations, we propose a novel error mitigation scheme, Pauli Check Extrapolation (PCE), which integrates PCS with an extrapolation technique similar to Zero-Noise Extrapolation (ZNE). However, instead of extrapolating to the 'zero-noise' limit, as is done in ZNE, PCE extrapolates to the 'maximum check' limit—the number of check pairs theoretically required to achieve unit fidelity. In this study, we focus on applying a linear model for extrapolation and also derive a more general exponential ansatz based on the Markovian error model. We demonstrate the effectiveness of PCE by using it to mitigate errors in the shadow estimation protocol, particularly for states prepared by the variational quantum eigensolver (VQE). Our results show that this method can achieve higher fidelities than the state-of-the-art Robust Shadow (RS) estimation scheme, while significantly reducing the number of required samples by eliminating the need for a calibration procedure. We validate these findings on both fully-connected topologies and simulated IBM hardware backends.**

***Index Terms*—quantum error mitigation, NISQ algorithms, hybrid quantum-classical architectures & computing**

I. INTRODUCTION AND BACKGROUND

This poster introduces Pauli Check Extrapolation (PCE) [1], a novel error mitigation scheme that combines Pauli Check Sandwiching (PCS) [2] with an extrapolation technique similar to Zero-Noise Extrapolation (ZNE) [3]. Our focus in this work is on mitigating errors in classical shadows [4] for states prepared by the variational quantum eigensolver (VQE). We show that PCE can achieve comparable or improved fidelities compared to the state-of-the-art Robust Shadow (RS) estimation scheme [5] while significantly reducing the sampling requirements.

Pauli Check Sandwiching (PCS) mitigates errors by inserting m pairs of controlled Pauli gates around the target circuit U. Each layer $L_m, R_m \in \mathcal{P}_n$ satisfies:

$$L_m U R_m = U.$$

An ancilla qubit is appended to the circuit for each check layer implementation. If all ancillas measure 0, the results are kept. Adding more layers allows for the detection of more errors, and according to Proposition 2 in [2], there is a theoretical maximum number of layers for which unit fidelity is achieved.

II. METHODOLOGY

The procedure for PCE is as follows:

1) Implement the first m layers of Pauli checks and measure the corresponding expectation values E_i.
2) Fit an extrapolation model $E(n)$ to the collected data (n, E_n). For instance, a linear model $E(n) = \alpha + \beta n$.
3) Use the fitted model to extrapolate the expectation value to the theoretical maximum number of check layers, estimating $E(n_{\text{max}})$.

III. RESULTS

For our experiments, we focus on error mitigating the classical shadow circuits for which a global Clifford unitary is

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appended. We can then protect either just the global Clifford portion, or include the entire circuit. These cases are referred to as 'check' for the global Clifford protection and 'prepcheck' for the entire circuit protection, as illustrated in Fig. 1. We present results for two state preparation circuits: a 4-qubit H_2 circuit and a 6-qubit H_2O circuit. Our comparison includes four scenarios: ideal noiseless ('noiseless'), unmitigated noisy ('noisy'), noisy with robust shadow estimation ('robust'), and noisy PCS with various numbers of layers, as well as noisy PCE ('extrap').

For each state preparation circuit, we generate $10,000$ shadow circuits and collect 100 measurement samples from each. Measurements for shadow tomography with Global Cliffords are done exclusively in the Z-basis. Accordingly, we use the first 3 check implementations to extrapolate to 4 checks for the H_2 circuit, and 4 checks to extrapolate to 6 checks for the H_2O circuit.

For the results shown in Fig. 1, a depolarizing error channel is used with two-qubit error rates of $p_2 = 0.02$ and single-qubit error rates of $p_1 = 0.002$, except for the simulated mock backend in Fig. 1b. In Fig. 1d, error rates follow a Gaussian distribution across qubits with standard deviations of 0.005 for two-qubit gates and 0.0005 for single-qubit gates.

In the 4-qubit experiments, the extrapolated check achieves higher fidelities than RS on both the fully connected device (Fig. 1a) and the Cairo device (Fig. 1b). Notably, the extrapolated checks for both the check and prep check achieve similar fidelities to the actual check implementation, suggesting that the extrapolated checks are accurately predicting the expectation values. For the 6-qubit experiments, better results are expected when protecting the entire circuit or parts of the state preparation, as suggested by the comparison in Fig. 1a. In Fig. 1d, we use an uneven distribution of error rates across qubits, showing one of PCE's advantages over RS, which assumes a uniform noise model. This is reflected in the improved performance compared to Fig. 1c.

IV. CONCLUSION AND FUTURE DIRECTIONS

In this poster, we introduced a novel error mitigation scheme. Future work includes assessing PCE's effectiveness in protecting both Clifford and partial state preparation circuits, especially with many non-Clifford gates. We also aim to explore PCE's scalability for larger circuits, optimize the extrapolation model, and test PCE on fully connected hardware for more practical insights.

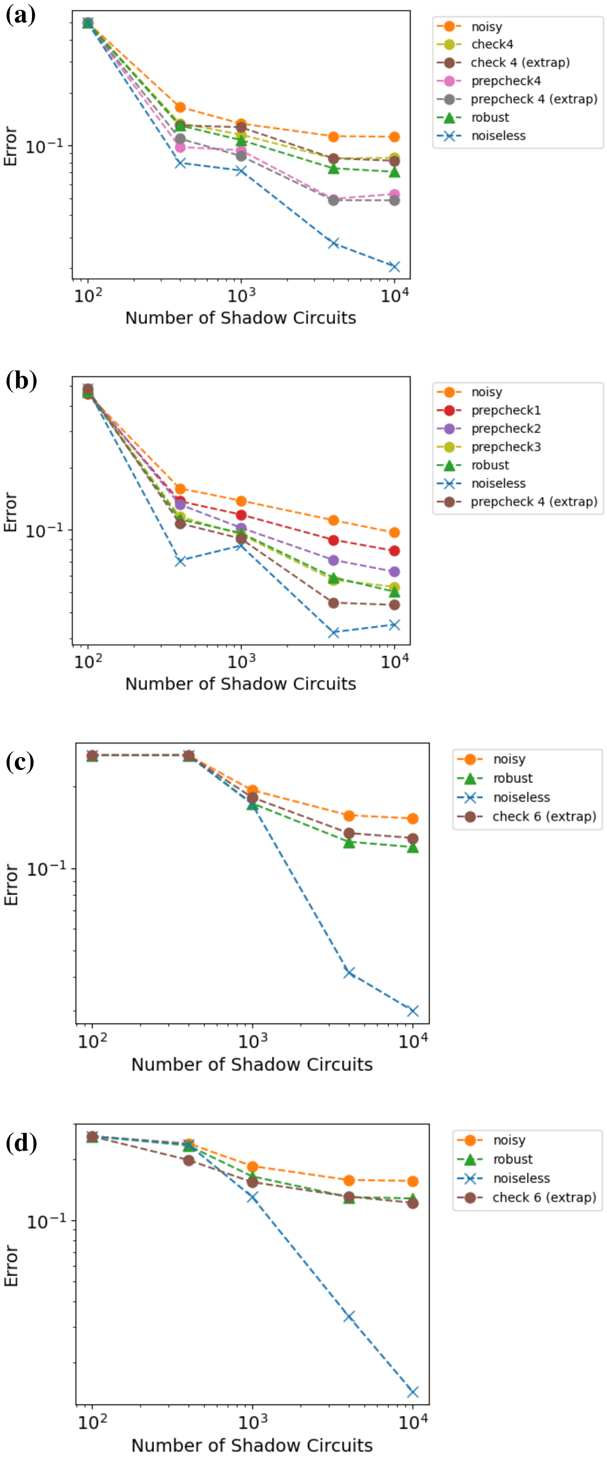

Fig. 1: (a) H_2 circuit with full connectivity. (b) H_2 circuit using the Cairo device backend. (c) H_2O circuit with full connectivity and constant error rates. (d) H_2O circuit with full connectivity and varying error rates.

REFERENCES

[1] Q. Langfitt, J. Liu, B. Huang, A. Gonzales, K. N. Smith, N. Hardavellas, and Z. H. Saleem, "Pauli check extrapolation for quantum error mitigation," 2024.

[2] A. Gonzales, R. Shaydulin, Z. H. Saleem, and M. Suchara, "Quantum error mitigation by pauli check sandwiching," *Scientific Reports*, vol. 13, Feb. 2023.

[3] K. Temme, S. Bravyi, and J. M. Gambetta, "Error mitigation for short-depth quantum circuits," *Phys. Rev. Lett.*, vol. 119, p. 180509, Nov 2017.

[4] H.-Y. Huang, R. Kueng, and J. Preskill, "Predicting many properties of a quantum system from very few measurements," *Nature Physics*, vol. 16, p. 1050–1057, June 2020.

[5] S. Chen, W. Yu, P. Zeng, and S. T. Flammia, "Robust shadow estimation," *PRX Quantum*, vol. 2, Sept. 2021.